Spatial Projections of Neural Arrays

A short guide to classic and new signal analysis techniques

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Abstract—**Electroencephalography and other neural recording techniques collect simultaneous data with a multitude of channels. A variety of methods have been proposed to analyze such high-dimensional data and go by various 3-letter acronyms such as PCA, ICA, LDA, SVM, CSP, DSS, CCA, CSD. What all of these methods have in common is that they integrate information by averaging across space, and the different techniques only differ in the contribution of each channel to the average. This has the potential to substantially improve signal quality. The goal of this presentation is to give an overview of existing techniques focusing on those techniques that have an easy to understand objective criterion. It should thus provide a guide on how to pick the technique that best suits a given experimental goal. The review will start with the simplest and most straightforward idea, and finish with a few more recent and novel techniques that are not yet widely known.**

To start, assume that a multidimensional signal $\boldsymbol{x}(t)$ is recorded and we wish to summarize this information into one or several "components" $y(t)$ that have a better signal-to-noise ratio (SNR) as compared to the original individual channels:

$$y(t) = \boldsymbol{w}^T \boldsymbol{x}(t) \quad . \tag{1}$$

The challenge lies in finding the optimal spatial projection vector $\boldsymbol{w}$ – or "weights" of the weighted sum in (1) – that will accomplish this task. Of course optimality will depend on what we consider signal and what is undesirable noise. The simplest possible idea would be that of "matched filtering" (introduced for EEG in [1]). Assume the signal of interest is the activity that is reliably evoked at time t_0 by an event, say, a button push or a flash on the screen. If we have several repetitions of the event, we would look at the average evoked response and consider that a "template" for the activity of interest. Matched filtering implies setting the projection vector equal to the template (over-bar indicating the mean value across repeats):

$$\boldsymbol{w} = \bar{\boldsymbol{x}}(t_o) \quad . \tag{2}$$

With this choice, channels that give a positive evoked response will contribute positively to the projection $y(t)$ and channels with a negative evoked response will *also* contribute positively. Therefore, regardless of the polarity of the evoked response on any one channel, all channels add up coherently in the new component $y(t)$ and channels that do not have an appreciable evoked response do not contribute. By construction, this approach will always give a positive projection at the time of interest, $y(t_0)>0$, even if one is looking at random noise. One can use standard randomized shuffling techniques to determine if this non-zero value rises above chance values. The same statement applies to all subsequent techniques discusses here. Note that matched filtering maximizes the mean value of the projection:

$$\max_w \bar{y}(t_0) \quad . \tag{3}$$

Typically, however, the experimenter is not just interested in the mean activity, but rather wants to know if the mean activity is large compared to the variation across repetitions in the experiment. That is, one may be interested in finding projections that maximize the mean over standard deviation across repetitions (omitting the dependence on time):

$$\max_w T = \max_w \sigma_{\bar{y}}^{-1} \bar{y} \quad . \tag{4}$$

This is the familiar T-statistic used for the Student's t-test. The projection that maximizes this statistic is simply given by:

$$\boldsymbol{w} = \boldsymbol{R}_{xx}^{-1} \bar{\boldsymbol{x}} \quad . \tag{5}$$

So, if one is interested in finding the linear combination of sensors that gives the largest t-statistic and thus the smallest p-value (probability of chance occurrence of an effect) all that is needed is the covariance $\boldsymbol{R}_{xx}$ and the mean $\bar{\boldsymbol{x}}$ across repeats measured at the time of interest. In many experiments one is not interested just in the activity evoked by one type of event, but rather, one would like to know if there is a difference between two experimental conditions. In that case the matched filter is given by the difference of the means:

$$\boldsymbol{w} = \bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2 \tag{6}$$

and the projection with the optimal T-statistic to detect this mean difference above the trial-to-trial noise is given by:

$$\boldsymbol{w} = \boldsymbol{R}_{xx}^{-1}(\bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2) \quad , \tag{7}$$

which is the well known Fisher Linear Discriminant [2]. To obtain robust results when taking the inverse of a covariance matrix it is standard procedure to remove outliers and/or apply shrinkage techniques in the computation of the covariance. Alternatively, one can use contemporary techniques that are inherently robust to outliers such as penalized logistic regression or support vector machines [1].

It may be that one is not interested in evoked responses, but rather, one would like to find activity that has maximum power, e.g., oscillations in a given frequency band without regard for the sign of the activity. In this case the phase is not important and instead just the strength of deviation from

978-1-4673-5974-0/13/$31.00 2013 IEEE

baseline matters. Mathematically, one would like to find the projection with the maximal standard deviation:

$$\max_{w} \ \sigma_{\tilde{y}}^2 = \max_{w} \ \boldsymbol{w}^T \boldsymbol{R}_{\tilde{x}\tilde{x}} \boldsymbol{w} \quad . \qquad (8)$$

(tilde ~ indicate that variance/covariance is be measured on filtered data). This criterion is optimized by the eigenvectors of the covariance matrix, or "principal components" [3]:

$$\boldsymbol{R}_{\tilde{x}\tilde{x}} \boldsymbol{w} = \lambda \boldsymbol{w} \quad . \qquad (9)$$

The eigenvector with the largest eigenvalue produces the component with the largest power. Subsequent eigenvectors are spatially orthogonal to the first and produce additional components that are uncorrelated from the first, and carry the next strongest power. Capturing components that are uncorrelated and capture successively the next strongest power may be useful (e.g. for dimensionality reductions), but spatial orthogonality is a meaningless constraint for most electrophysiological recordings. In addition, the power in a specific frequency band may in-itself not be particularly meaningful. More typically one is interested in the power within a frequency band relative to the total power in the signal. That is, one may wish to maximize:

$$\max_{w} \frac{\sigma_{\tilde{y}}^2}{\sigma_v^2} = \max_{w} \frac{\boldsymbol{w}^T \boldsymbol{R}_{\tilde{x}\tilde{x}} \boldsymbol{w}}{\boldsymbol{w}^T \boldsymbol{R}_{xx} \boldsymbol{w}} \quad . \qquad (10)$$

The solutions are provided by this eigenvalue equation:

$$\boldsymbol{R}_{xx}^{-1} \boldsymbol{R}_{\tilde{x}\tilde{x}} \boldsymbol{w} = \lambda \boldsymbol{w} \quad . \qquad (11)$$

The eigenvector in this equation with the strongest eigenvalue generates the linear projection or component of the data with the highest SNR – assuming uncorrelated noise, optimizing (10) is equivalent to optimizing SNR. The eigenvector with the second highest eigenvalue generates a component that is uncorrelated from the first and captures the next highest SNR, etc. Note that these projections vectors are no longer orthogonal. Instead, eigenvalue equation (11) ensures that the components are also uncorrelated for the filtered data. This additional condition allows us to drop the physiologically meaningless spatial orthogonality constraint. The approach of (11) is is the same as in Common Spatial Patterns, where the two covariance matrices are measured on the original data, but in two separate periods of time (see [1]). This will generate components sorted by the power-ratio between those two time periods, which is useful to detect components that exhibit a maximum change in power, e.g., alpha "desynchronization" [1]. Various other choices for the matrices result in a number of different blind source separation techniques [4]. This version presented here in particular represents a special case of linear Denoising Sources Separation [5].

As before, one may be worried about the numerical robustness of the eigenvalue equation (11) – in particular if the two covariances are very similar. This may happen when looking for signals with very small SNR, or, equivalently, the effect size is small. In that case, a linear approximation of the power-ratio criterion above is useful, namely, the difference of the two covariances, e.g., measured at times t_1 and t_2:

$$\max_{w} \ \left(\sigma_y^2(t_1) - \sigma_y^2(t_2) \right) \quad , \qquad (12)$$

Support for this work was provided in part by DARPA/DSO NIA and N2 programs.

This is optimized by the eigenvectors of the covariance differences:

$$\left(\boldsymbol{R}_{xx}(t_1) - \boldsymbol{R}_{xx}(t_2) \right) \boldsymbol{w} = \lambda \boldsymbol{w} \quad . \qquad (13)$$

We have used this successfully to extract very small variations in the alpha power during free viewing visual search (Dias et al., in preparation).

We would like to mention one last approach for extracting linear projections of interest. Sometimes there are no well-defined events in the experimental paradigm that can serve as time markers to facilitate averaging over repeated events. Imagine the experimental subjects are exposed to a continuous stimulus, say, human subjects watching a movie. Instead of regressing the data against discrete events in time, one can then instead regress them against the signal recorded from a different subject who was exposed to the identical video (or a repetition with the same subject). In that case one is interested in the component that maximizes the correlation between signals from different subjects/repeats, i.e., we now want to maximize the correlation coefficient between data-set 1 and 2. After symmetrizing the problem this can be formulated as:

$$\max_{w} \ cc(y_1, y_2) = \max_{w} \frac{\boldsymbol{w}^T \left(R_{x_1 x_2} + R_{x_2 x_1} \right) \boldsymbol{w}}{\boldsymbol{w}^T \left(R_{x_1 x_1} + R_{x_2 x_2} \right) \boldsymbol{w}} \qquad (14)$$

Again, mathematically the answer is the same as before, namely, eigenvectors of the following eigenvalue problem:

$$\left(R_{x_1 x_1} + R_{x_2 x_2} \right)^{-1} \left(R_{x_1 x_2} + R_{x_2 x_1} \right) \boldsymbol{w} = \lambda \boldsymbol{w} \quad . \qquad (14)$$

This approach which we call "correlated component analysis" [6], differs from conventional canonical correlation analysis (CCA) [7] in two important ways:1) by using the same projections vectors for both data sets the dimensionality of the problem is reduced. 2) Reducing the number of unknowns allows one to dispense of the spatial orthogonality constraint of CCA , which, as stated before, is meaningless in most physiological recordings. With this approach we have found EEG components that are indicative of subjective engagement in a movie clip with a second-by-second resolution [6].

A extended version of this short guide is in preparation. Please contact LCP if you would like to reference this material.

References

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